On some open points in modelling and model predictive control

Control in Steel,

The Future of Control in the Steel Sector

Webinar, 14.07.2022

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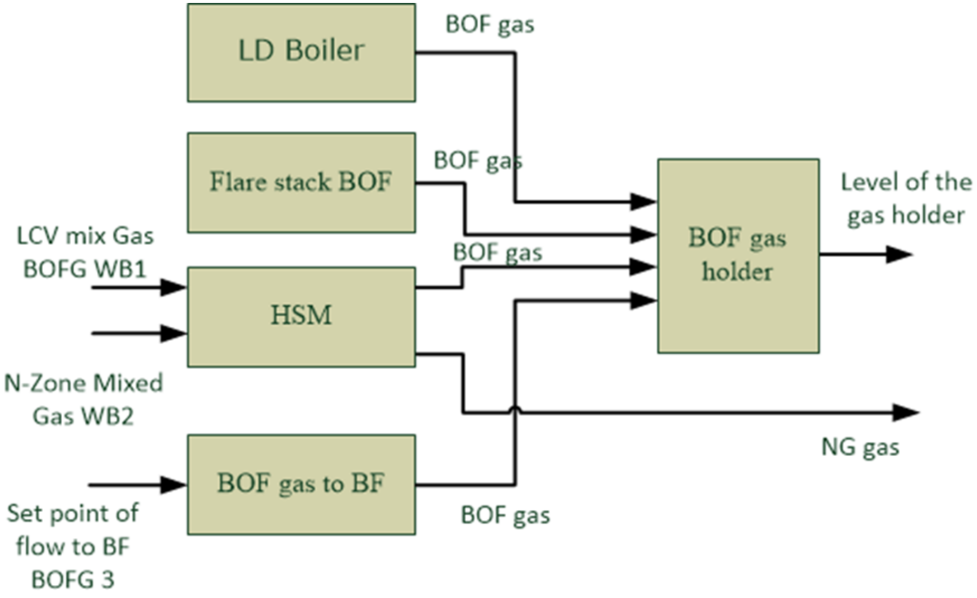
Research Fund for Coal & Steel
ESTEP
Stahl

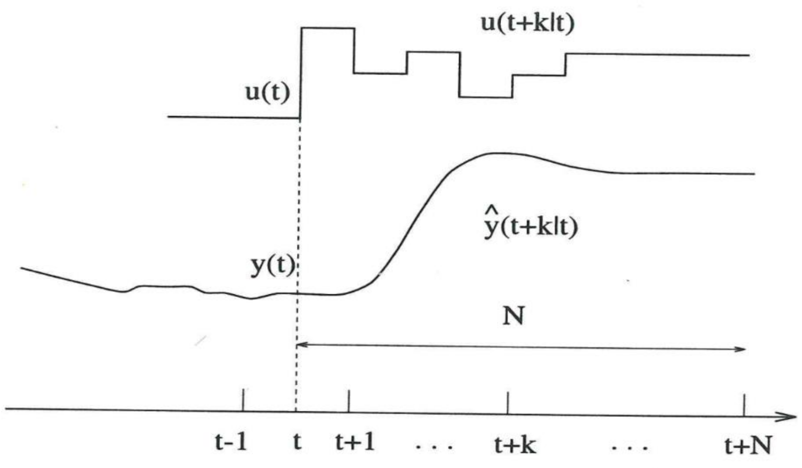
Agenda

1. On some problems of mixed-integer optimisation

2. Safe Reinforcement Learning

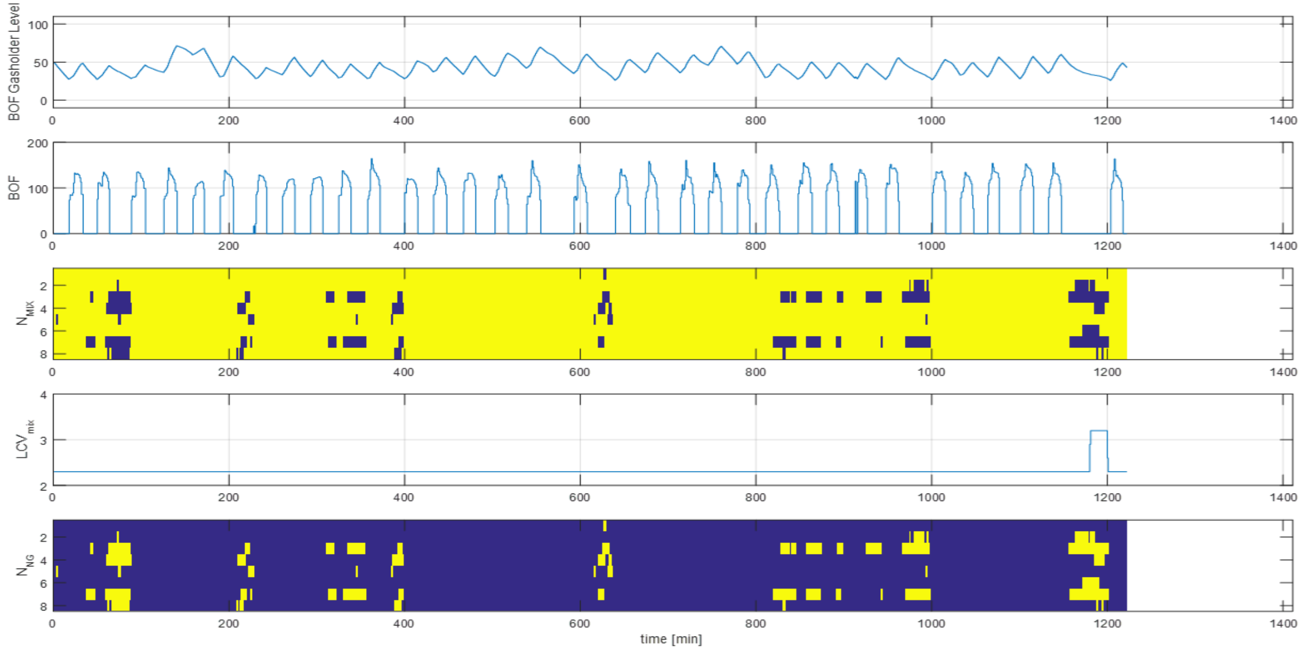
Example: Economic optimisation of BOF gas supply





Cost function

$$\text{minimize } J = \sum_{k=0}^{N} \gamma^k \left(l_{holder}(k) + l_{NG}(k) + l_{zone}(k) + l_{flare}(k) + l_{BOF\ to\ BF}(k) \right)$$



Example: Economic optimisation of BOF gas supply

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Bilinear mixed integer system

Mixing station

- $\dot{V}_{BOF}(k) = (1 - x_{LCV}(k))\dot{V}_{MIX}(k)$
- $\dot{V}_{NG}(k) = x_{LCV}(k)\dot{V}_{MIX}(k)$
- $x_{LCV}(k) = \frac{LCV_{mix}(k) - LCV_{BOF}(k)}{LCV_{NG}(k) - LCV_{BOF}(k)}$

HSM Control Model

- $\dot{V}_{mix}(k) = \sum_{i=1}^{NZone} \delta_{zone,i}(k)\,\dot{V}_{MIX,zone,i}(k)$
- $\dot{V}_{NG}(k) = \sum_{i=1}^{NZone} \delta_{NG,i}(k)\,\dot{V}_{NG,zone,i}(k)$
- $\delta_{zone,i}(k) + \delta_{NG,i}(k) = 1$
- $\dot{V}_{MIX,zone,i}(k) = \frac{1}{LCV_{MIx}(k)} E_{zone,i}(k, \ldots..)$
- $\dot{V}_{NG,zone,i}(\mathrm{k}) = \frac{1}{LCV_{NG}(k)} E_{zone,i}(k, \ldots \ldots)$

Bilenar Systems

- $\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k + \sum_{i=1}^{m} C_i u_{i.k}$

Discretisation of the input variables

- $\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{D}\boldsymbol{\delta}_k + \boldsymbol{\delta}'_k\boldsymbol{D}'\boldsymbol{C}\boldsymbol{x}_k$
- with
- $\boldsymbol{u}_k = \boldsymbol{D}\boldsymbol{\delta}_k,\ \boldsymbol{D} \triangleq u_0[2^0 \quad \ldots \quad 2^{r-1}]$

Big-M Approach

- $y_k = \delta_k \mathrm{f}(x_k)$
- $y_k \le M\delta_k$
- $y_k \ge m\delta_k$
- $y_k \le f(x_k) - m(1 - \delta_k)$
- $y_k \ge f(x_k) - M(1 - \delta_k)$

Example: Economic optimisation of BOF gas supply

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- $\dot{V}_{BOF}(k) = \left(1 - x_{LCV}(k)\right)\dot{V}_{MIX}(k)$
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Bilinear mixed integer system

- The transformation into a linear mixed-integer system using the Big M approach leads to a high number of logical decision variables
- In this case

  Horizon = 2 h

  Logical Variables: 7920

  Continuous Variables: 360

  Constraints: 23208
- Can it be simplified and accelerated

Example of an Multi Linear System: Heating system

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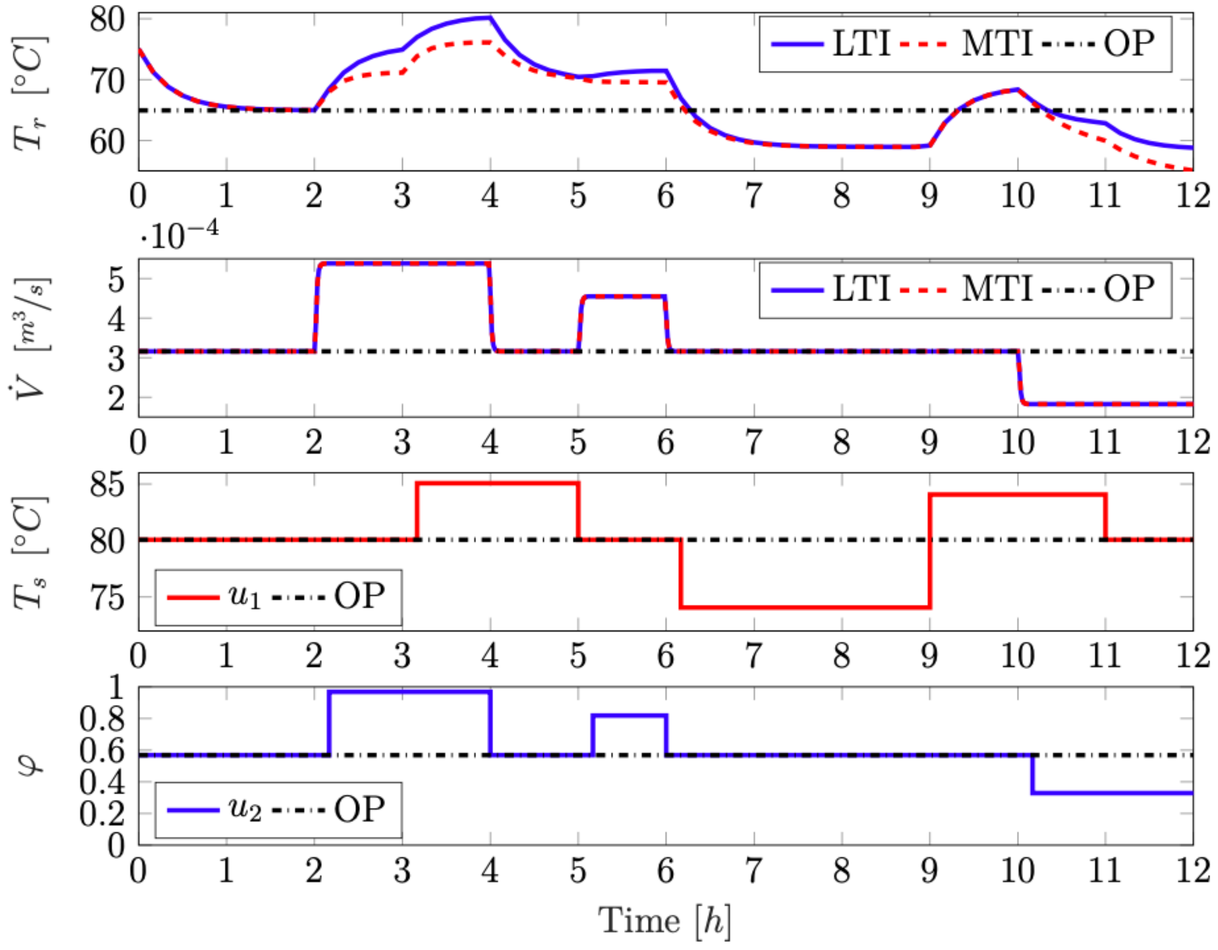
$$\ddot{V} = -\frac{1}{\tau_{\dot{V}}}\dot{V} + \frac{\dot{V}_{max}}{\tau_{\dot{V}}}\varphi,$$

$$c\rho V_{rad}\dot{T}_r = c\rho\dot{V}T_s - c\rho\dot{V}T_r - \dot{Q}_{heat},$$

$$\dot{x}_1 = p_1\left(u_1 x_2 - x_1 x_2\right) + p_2,$$
$$\dot{x}_2 = p_3 x_2 + p_4 u_2.$$



Kai Kruppa: Phd-Theses:Multilinear Design of Decentralized Controller Networks for Building Automation Systems

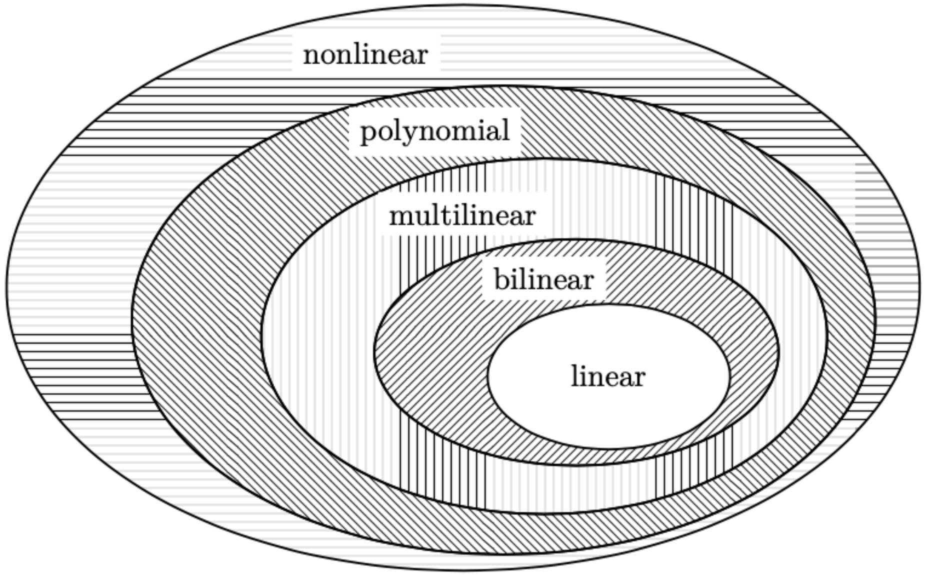
Observation

- Approximation of bilinear systems by discretisation and Big-M approach leads to very many decision variables.

- Mixed integer for a non-linear process requires a lot of computing time.

Is there another approach ?

- Many procedural processes can be described as multi-linear systems and can be sufficiently approximated in the same way.



Boolesche Algebra

Zhegalikin Polynome

Boolean function	algebraic function
NOT x_1	$1 - x_1$
x_1 AND x_2	$x_1 x_2$
x_1 OR x_2	$x_1 + x_2 - x_1 x_2$

$\underline{b}_1$	$\underline{b}_2$	XOR$(\underline{b}_1, \underline{b}_2)$
0	0	0
0	1	1
1	0	1
1	1	0

$$\underline{f}_b(\underline{b}_1, \underline{b}_2) = \underline{b}_1 + \underline{b}_2 - 2\underline{b}_1\underline{b}_2.$$

Boolean state space models

$$\begin{aligned} \underline{\mathbf{x}}(k+1) &= \underline{\mathbf{f}}(\underline{\mathbf{x}}(k)\underline{\mathbf{u}}(k)), \\ \underline{\mathbf{y}}(k) &= \underline{\mathbf{f}}(\underline{\mathbf{x}}(k)\underline{\mathbf{u}}(k)), \\ \underline{\mathbf{x}}(0) &= \underline{\mathbf{x}}_0, \end{aligned}$$

Zhegalikin polynomials are multilinear

Saadia Faisal; Gerwald Lichtenberg; Herbert Werner (2008). *Polynomial models of gene dynamics. , 71(13-15), 2711–2719.* doi:10.1016/j.neucom.2007.09.024

Struktur of multi-linear hybrid model

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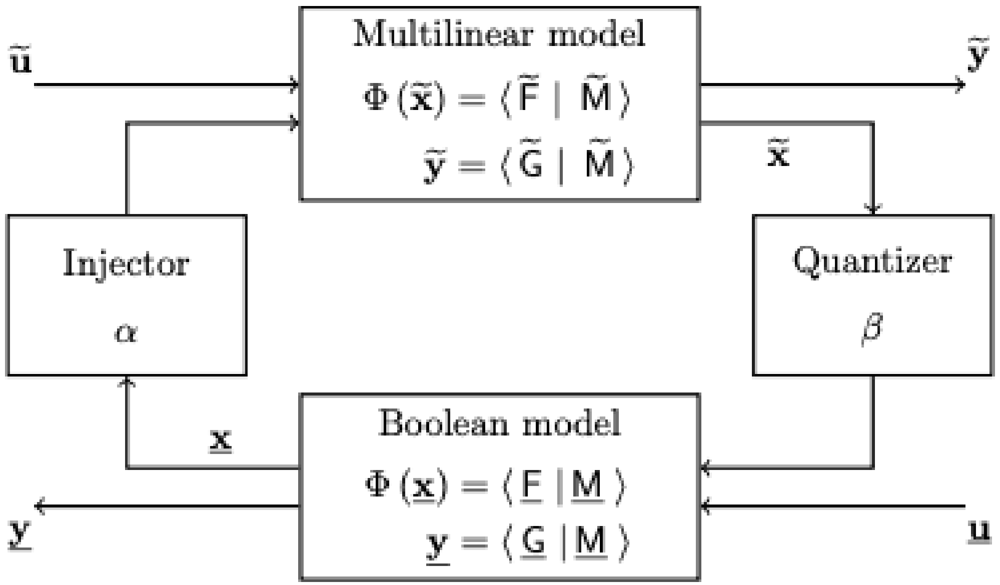
$$(\alpha(\mathbf{x}))_i = \begin{cases} 1 \in \mathbb{R} & \textit{if } x_i = \text{TRUE}\,, \\ 0 \in \mathbb{R} & \textit{if } x_i = \text{FALSE}\,, \\ x_i & \textit{if } x_i \in \mathbb{R}\,. \end{cases}$$



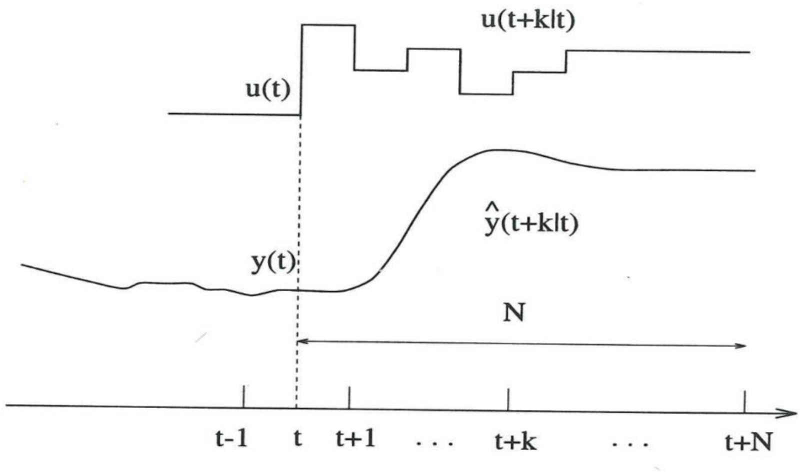
$$(\beta(\mathbf{x}))_i = \sigma(x_i - \frac{1}{2})\,,$$

$$\sigma(x) = \begin{cases} 1 & \textit{if } x \geq 0\,, \\ 0 & \textit{otherwise}\,. \end{cases}$$

Relaxing mixed integer economic optimisation

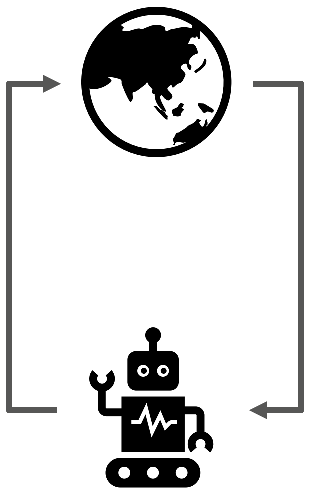
$$J_{x,b}(\mathbf{X}, \underset{\sim}{\mathbf{U}}) = J_x(\mathbf{X}, \underset{\sim}{\mathbf{U}}) + J_b(\underset{\sim}{\mathbf{U}}) \,.$$

$$\begin{aligned} \underset{\sim}{\mathbf{x}}_a(k+1) &= \underset{\sim}{\mathbf{F}}_a \mathbf{L}(\underset{\sim}{\mathbf{x}}_a(k), \underline{\mathbf{z}}(k)) \,, \\ \underset{\sim}{\mathbf{u}}(k) &= \underline{\mathbf{G}}_a \mathbf{L}(\underset{\sim}{\mathbf{x}}_a(k), \underline{\mathbf{y}}(k))), \end{aligned}$$



Reinforcement learning

Action u_k



State x_k

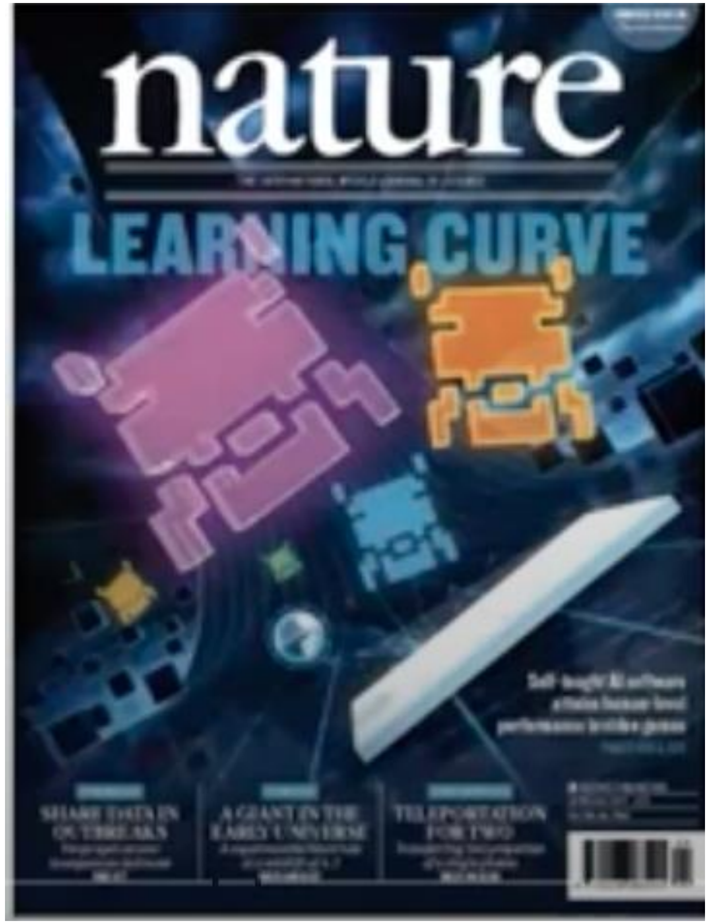
Reward r_k

Successful in the field of computer games
Alpha GO

Connection to adaptive control

Is it safe in industriell environment?

Reinforcement Learning beyond the simulated environment



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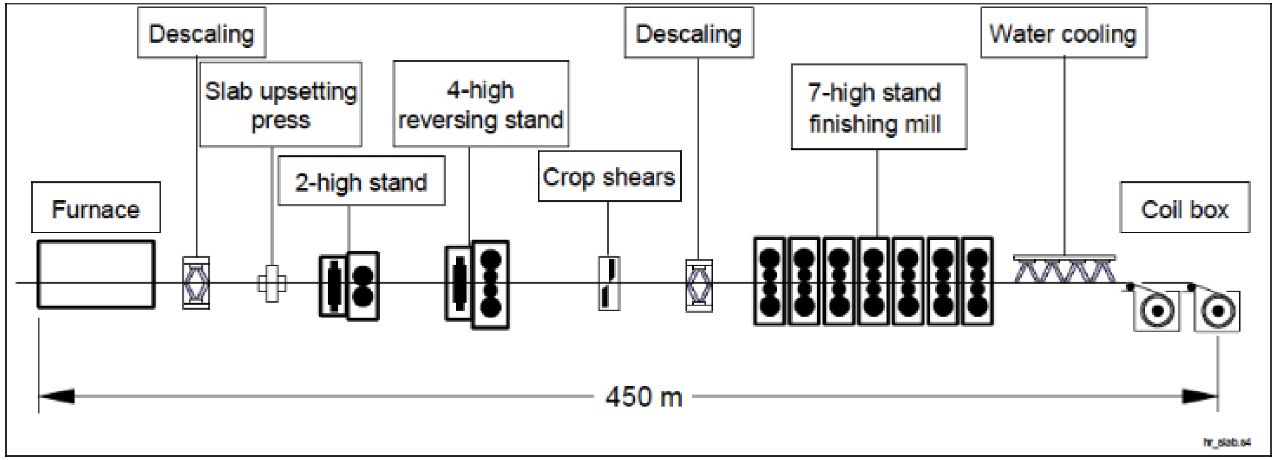
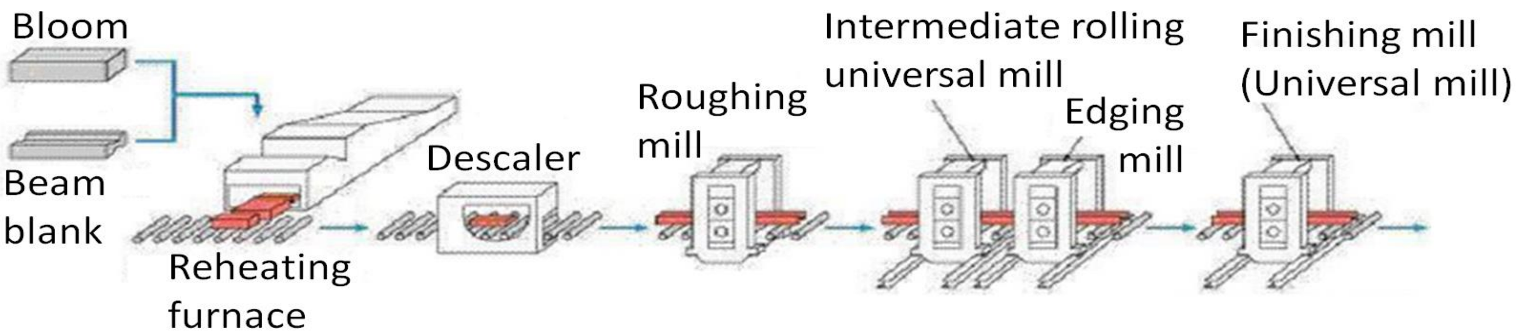
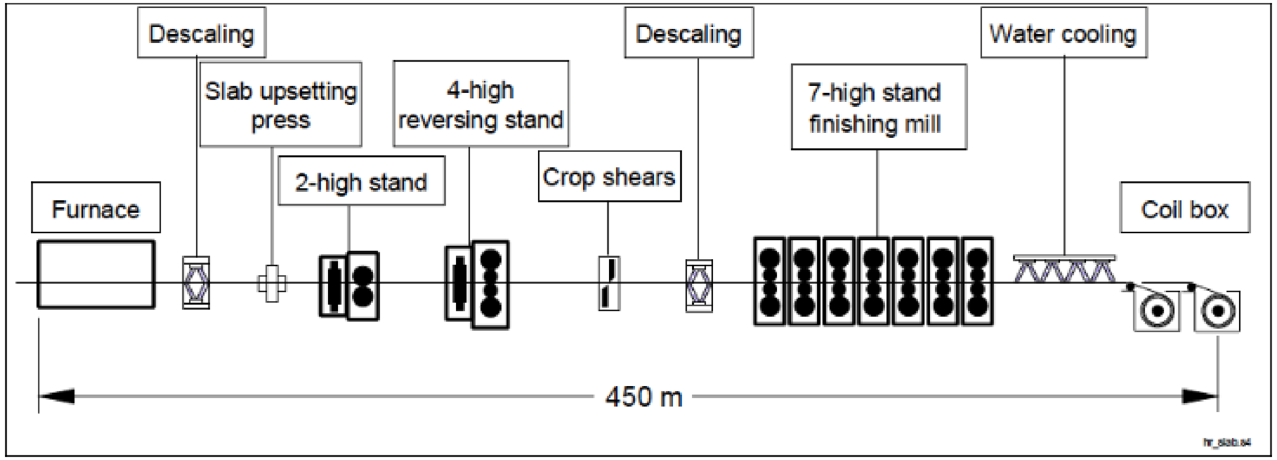
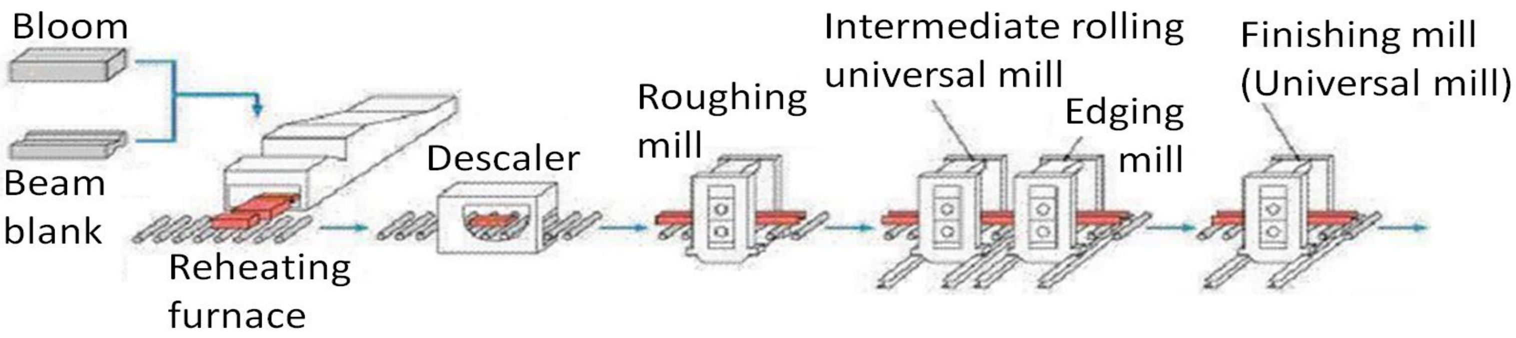




Reinforcement learning beyond the simulated environment

- How can a learning system explore efficiently while ensuring safety?





Safe optimisation

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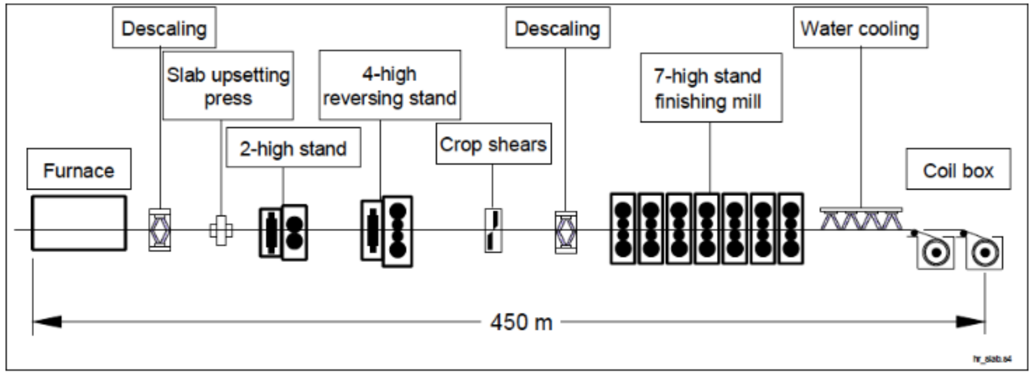
u_k



$f(u_k) + \epsilon_k^f$

Reward noisy

$g(u_r) + \epsilon_k^g$

Constraint noisy

unknown

Goal: $\min_{u_k} f(u_k) \;\; s.t. \;\; g(u_k) \geq 0$

Safety: $g(u_k) \geq 0 \; for \; all \; k$

see: Andreas Krause ETH Zürich : Safe and Efficient Exploration in Reinforcement Learning
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Bayesian Optimization

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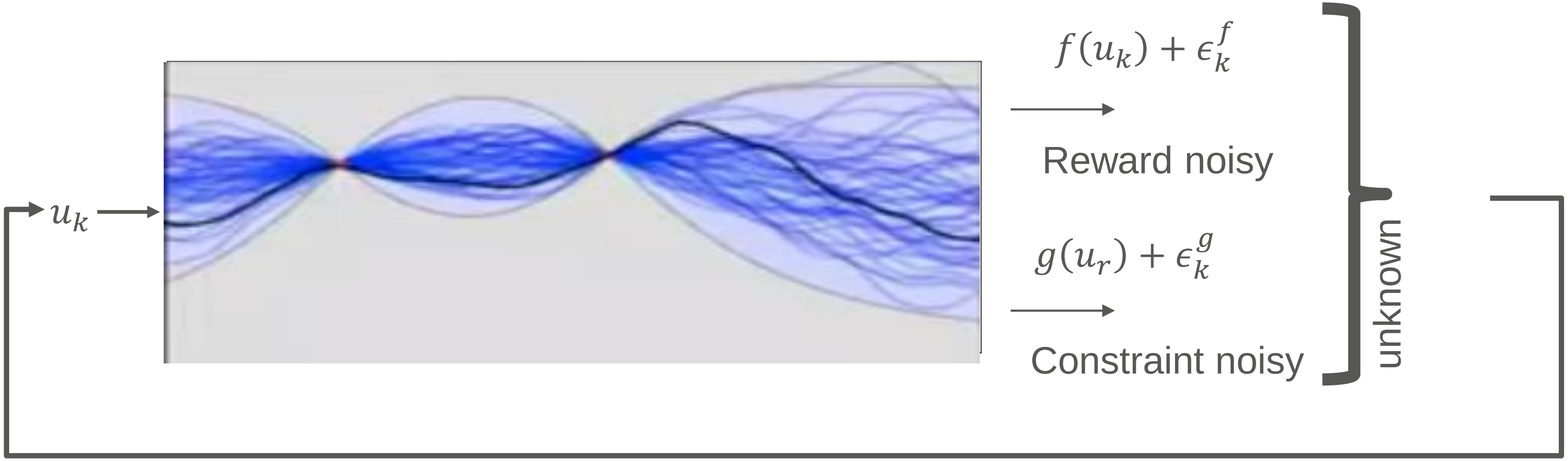

Goal: $\min_{u_k} f(u_k) \;\; s.t. \;\; g(u_k) \geq 0$

Safety: $g(u_k) \geq 0 \; for \; all \; k$

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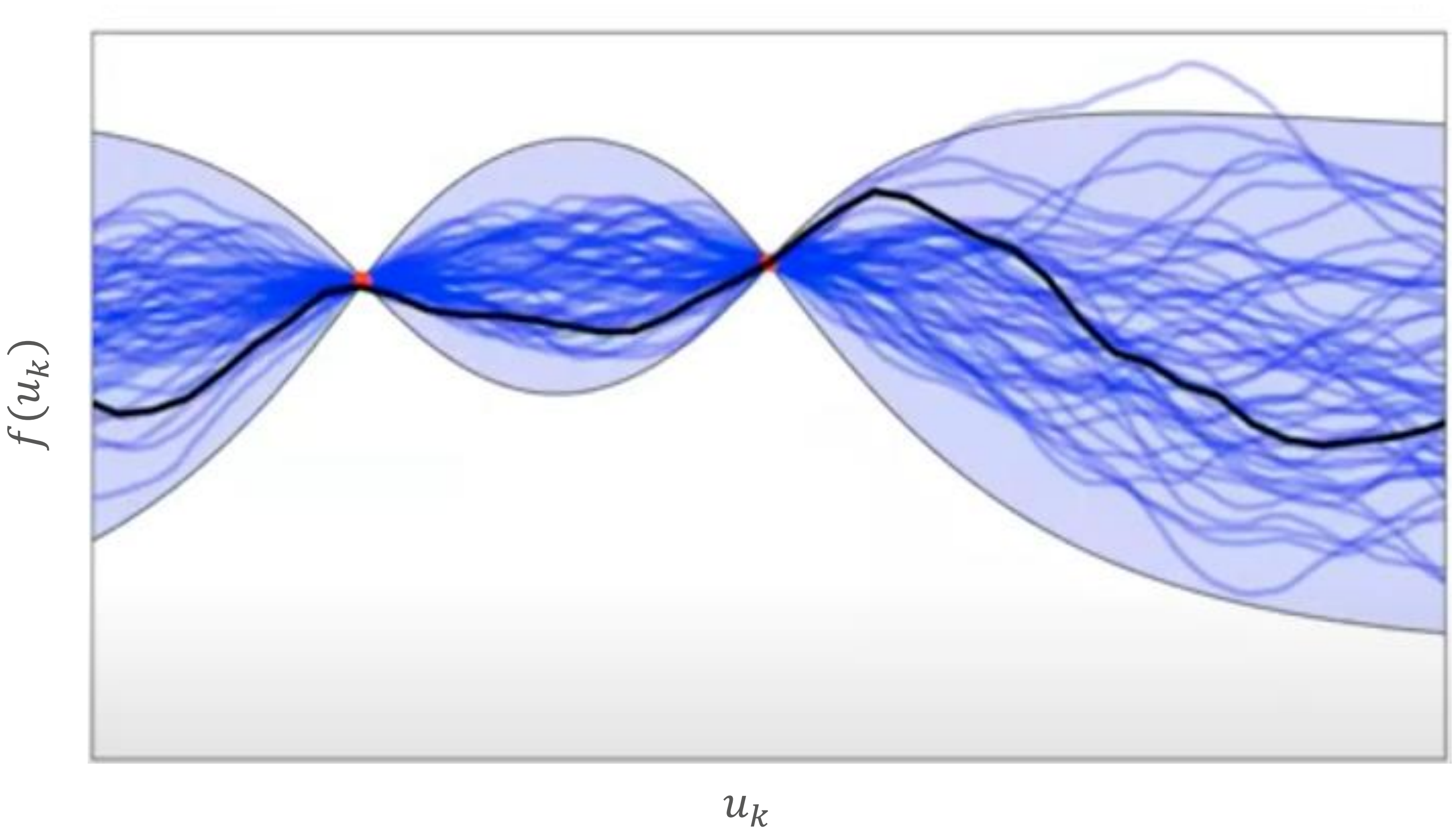
Gaussian regression models



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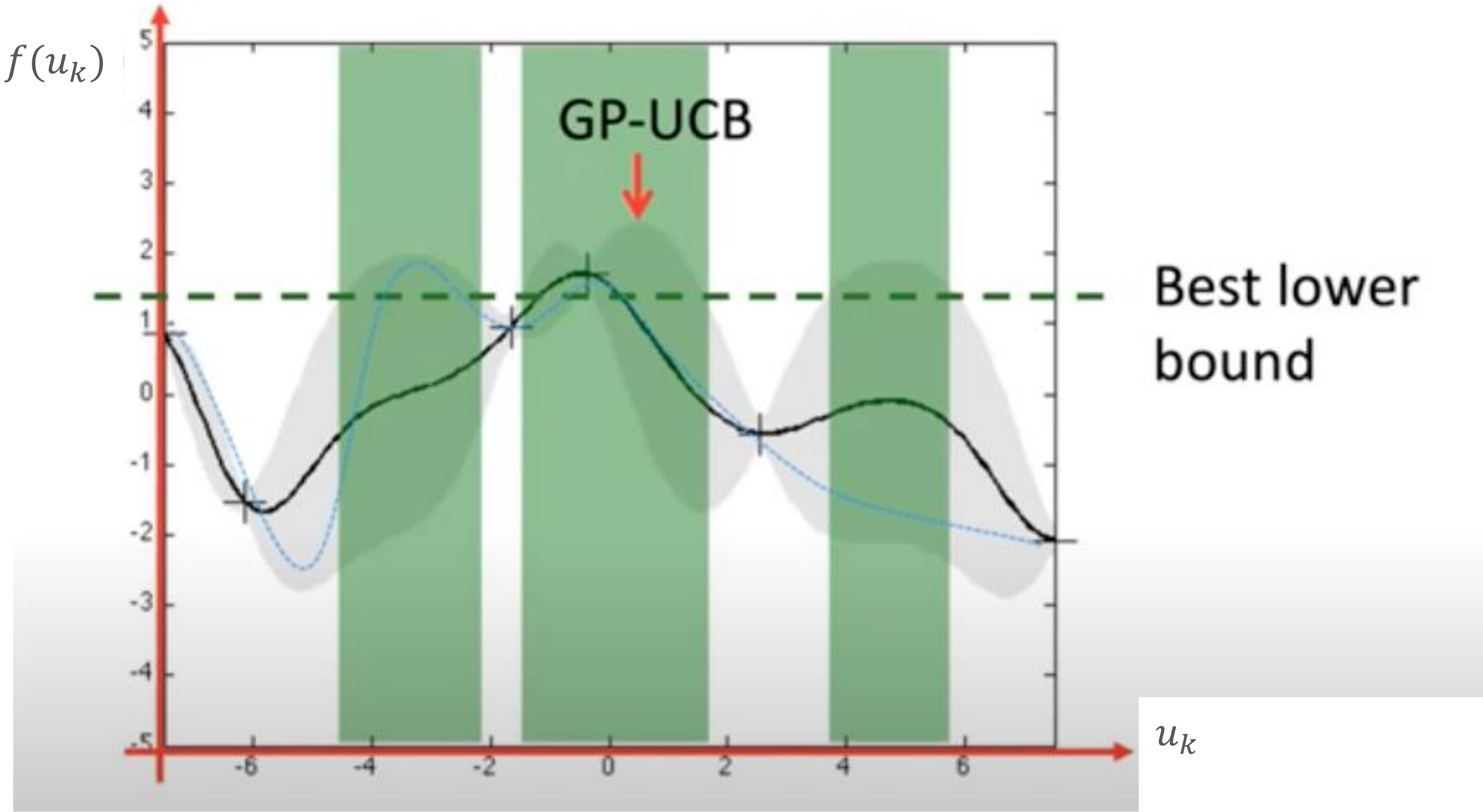
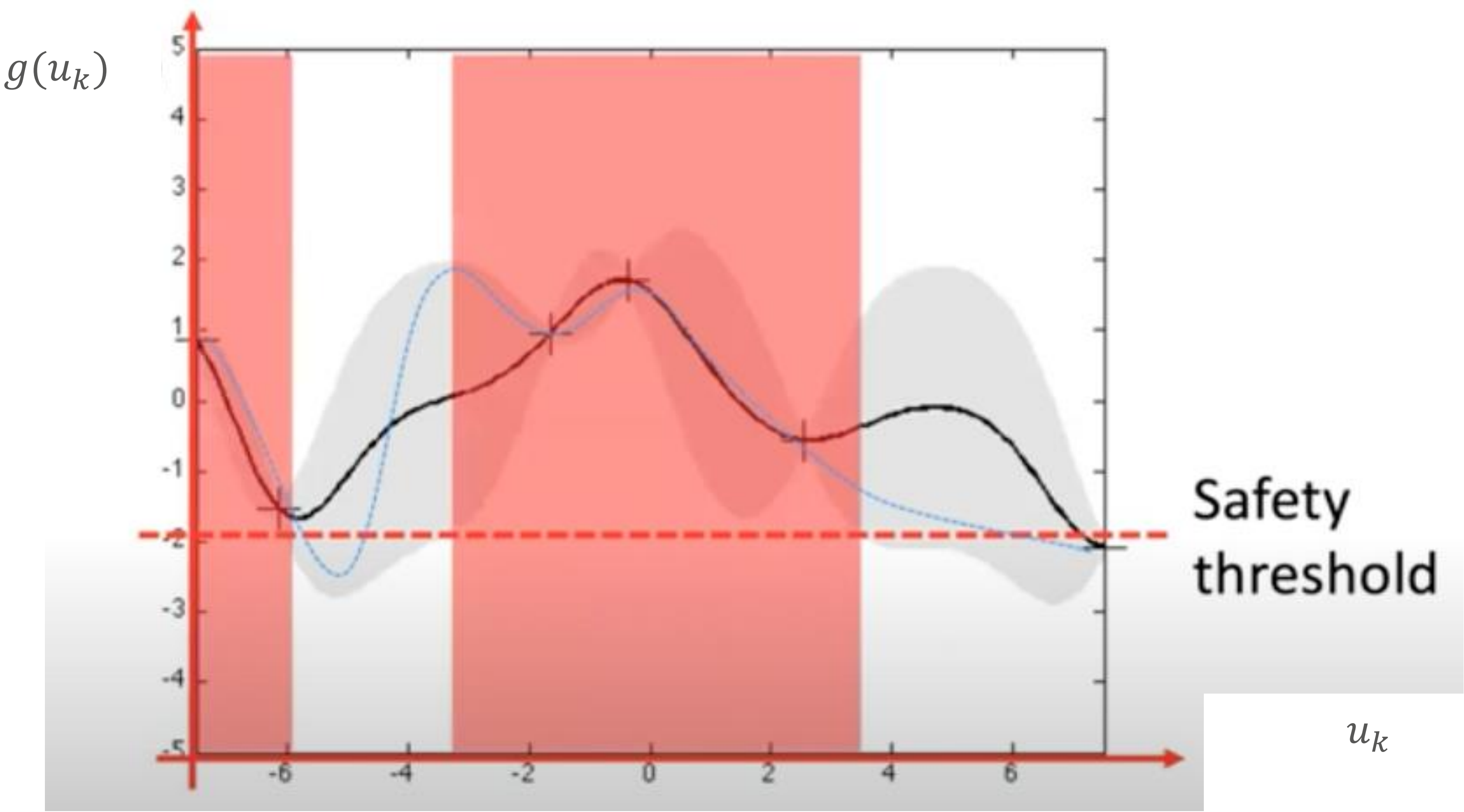
Bayesian Optimization



Bayesian Optimization



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Thank you for your attention!

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